A Novel Model for Reverberant Signals; Robust Maximum Likelihood Localization of Real Signals Based on a Sub-Gaussian Model

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Abstract

In this paper we present a novel model for signals encountered in reverberant environments using the sub-Gaussian distribution that can describe both the impulsive nature of the signals and their inter-dependence. The proposed system can be viewed as one where the sources are stochastic and Gaussian and the transfer medium is varying in a highly impulsive manner, introducing the sub-Gaussian nature at the receiver or alternatively, the impulsive transformation to the signals can be viewed as part of the source model, creating a multivariate source signal whose components can not be independent, and is of impulsiveness equal to the one of the Cauchy distribution.

We formulate the separable Maximum Likelihood solution to an array signal processing problem based on a derived sub-Gaussian density. We proceed to give both simulations and experimental results of the validity of the algorithm. In the experiments sound signals are played from loudspeakers in a room and localized with a microphone array and it is demonstrated that the localization based on the sub-Gaussian model significantly outperforms the one based on the traditional Gaussian model.

1 Introduction

The Gaussian distribution has traditionally been the most widely accepted distribution and used, as a rule, as a realistic model for various kinds of noise. In recent years however, there has been a tremendous interest in the class of α-stable processes, which are a generalization of the Gaussian process, but are able to model a wider range of phenomena and can be of a more impulsive nature. In fact, the Gaussian is the least impulsive α-stable distribution, while other widely known distributions of this class are the Cauchy and the Lévy.

These authors have demonstrated in previous work [1] the appropriateness of the α-stable processes for modeling noises encountered in audio environments and presented a time delay estimation method for localization of speech sources. Additionally, it is well known that the reverberant components of audio signals are highly dependent on the original source.

In this paper we start first by developing a model that is able to account for both the dependence and impulsive nature of the signals and we proceed to derive the density function of the proposed model. Based on this density function, we formulate an array signal processing problem in section 3, and its *Maximum Likelihood* (ML) estimator in 4. To reduce both computational complexity and numerical errors the separable solution of the ML is derived in section 5. Subsequently, in order to demonstrate the advantages of the proposed model some simulations comparing the Gaussian based ML and the sub-Gaussian based ML are given in section 6. The significant real world advantages of this novel model are then demonstrated with experimental results in section 7.

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2 Sub-Gaussian Random Variables

Sub-Gaussian distributions are a special case of α-stable random processes. A Sub-Gaussian random vector $\boldsymbol{X}$ can be defined as a random vector with characteristic function of the form

$$\varphi(\boldsymbol{u}) = \exp\left(-\frac{1}{2}\left[\boldsymbol{u}^T \mathrm{R}\, \boldsymbol{u}\right]^{\alpha/2}\right) \tag{1}$$

where R is a positive-definite matrix. Sub-Gaussian processes are variance mixtures of Gaussian processes [2].

If $\boldsymbol{X}(t)$ is sub-Gaussian with parameter α (will be denoted by α-SG(R)) and S is a positive stable process with characteristic exponent $\alpha/2$ (*i.e.*, S is $\frac{\alpha}{2}$-stable random variable completely skewed to the right) and $\boldsymbol{Y}(t)$ is a multivariate Gaussian process independent of S, then:

$$\boldsymbol{X}(t) = S^{1/2}\boldsymbol{Y}(t) \tag{2}$$

More information on sub-Gaussian random processes can be found in [3].

The proposed model for audio signals, based on sub-Gaussian processes, utilizes the stable positive Lévy [4] process of equation (3) together with the multivariate-Gaussian of (4). The Lévy distribution of (3) – also referred to as a Paretto type 5 distribution – has an index of symmetry $\beta = 1$ and a characteristic exponent $\alpha = 0.5$, and thus should result in a sub-Gaussian density of equal impulsiveness to the Cauchy, but with the additional restriction of dependent components.

$$f(u) = \begin{cases} \frac{u^{-\frac{3}{2}} e^{-\frac{1}{4u}}}{2\sqrt{\pi}} & \text{if } u > 0 \\ 0 & \text{if } u < 0 \end{cases} \tag{3}$$

$$f(\mathrm{V}) = \prod_{f=f_1}^{f_M} \frac{1}{\pi^{\kappa}|\Sigma|} \exp\left(-\boldsymbol{v}^{\dagger}(f)\Sigma^{-1}\boldsymbol{v}(f)\right) \tag{4}$$

From the definition of sub-Gaussian processes (2), the signal $\boldsymbol{s} = [s_1 \dots s_\kappa]^{\mathrm{T}}$ is of the form

$$s_k(t) = u_k(t)^{\frac{1}{2}} \cdot \boldsymbol{v}_k(t) = w_k(t) \cdot \boldsymbol{v}_k(t) \tag{5}$$

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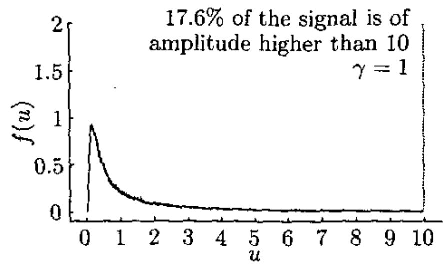

Figure 1: Lévy distribution and histogram of the data.

In order to find the density of $s_k(t)$, we first need the density of $w_k(t)$. Following the procedure outlined in [5]

$$f(w) = \begin{cases} \frac{w^{-2} e^{-\frac{1}{4w^2}}}{\sqrt{\pi}} & \text{if } w > 0 \\ 0 & \text{if } w < 0 \end{cases} \tag{6}$$

The distribution of the overall transmitted signal can be given by the multivariate distribution function:

$$\begin{aligned} F(\mathrm{S}) &= \int_{w=-\infty}^{+\infty} \int_{\boldsymbol{v}=-\infty}^{\boldsymbol{s}/w} f(w)\, f(\boldsymbol{v})\, d\boldsymbol{v}\, dw \\ &= \int_{w=0}^{+\infty} \int_{\boldsymbol{v}=-\infty}^{\boldsymbol{s}/w} \frac{w^{-2} e^{-\frac{1}{4w^2}}}{\sqrt{\pi}} \frac{1}{\pi^{\kappa}\,|\Sigma|} \\ &\quad \cdot \exp\left(-\boldsymbol{v}^{\dagger}(t)\Sigma^{-1}\boldsymbol{v}(t)\right)\, d\boldsymbol{v}\, dw \end{aligned} \tag{7}$$

Differentiating with respect to $\boldsymbol{s}$, and then integrating with respect to w

$$\begin{aligned} f(\boldsymbol{s}) &= \int_{w=0}^{+\infty} \frac{d}{dv} \int_{\boldsymbol{v}=-\infty}^{\boldsymbol{s}/w} \frac{w^{-2} e^{-\frac{1}{4w^2}}}{\sqrt{\pi}} \frac{1}{\pi^{\kappa}\,|\Sigma|} \\ &\quad \cdot \exp\left(-\boldsymbol{v}^{\dagger}(t)\Sigma^{-1}\boldsymbol{v}(t)\right)\, d\boldsymbol{v} \left(\frac{1}{\frac{ds}{d\boldsymbol{v}}}\right) dw \\ &= \int_{w=0}^{+\infty} \frac{w^{-2} e^{-\frac{1}{4w^2}}}{\sqrt{\pi}} \frac{1}{\pi^{\kappa}\,|\Sigma|} \\ &\quad \cdot \exp\left(-\boldsymbol{s}^{\dagger}(t)\Sigma^{-1}\boldsymbol{s}(t)/w^2\right) \cdot w^{-1}\, dw \end{aligned} \tag{8}$$

Therefore

$$f(\boldsymbol{s}) = \frac{1}{2\sqrt{\pi}\,\pi^{\kappa}\,|\Sigma|} \cdot \left[{}^{1}\!/_{4} + \boldsymbol{s}^{\dagger}(t)\Sigma^{-1}\boldsymbol{s}(t)\right]^{-1} \tag{9}$$

In the above Σ is the covariance matrix of the underlying Gaussian process. Additionally note that if the Gaussian random variable was one dimensional and real, then:

$$f(\boldsymbol{s}) = \frac{1}{2\sqrt{2}\pi\sigma} \cdot \left[{}^{1}\!/_{4} + \frac{s^2}{2\sigma^2}\right]^{-1} \tag{10}$$

which as theory predicts is the Cauchy pdf.

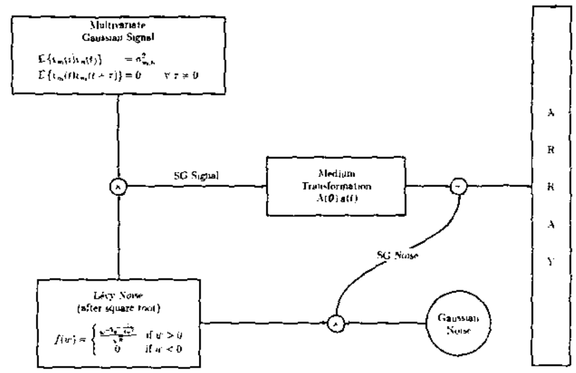

Figure 2: A multivariate Gaussian signal, corrupted by multiplicative Lévy noise, is then transformed through a set of delays to the receiving end of the array. The additive noise can be generated from the same Lévy process to make it jointly sub-Gaussian with the signal.

3 Array Problem Formulation

We assume that the transmitted signals are stochastic, and as such, the parameters of interest will be their statistics and *Directions-of-Arrival* (DOA's), and the sources are to be localized using a Maximum Likelihood estimator. We also assume a scenario under which there are $k = 1 \ldots \kappa$ sources received by an array of $r = 1 \ldots \rho$ sensors. The transfer function each signal undergoes while traveling to the array can be modeled as an attenuation and a delay. The attenuation will be considered the same at all sensors under the assumption that the sources are in the far field of the array, and with the additional assumption of a linear array:

$$a_{r,k} = \mathrm{e}^{-\mathrm{i}\omega\tau_{r,k}} = \mathrm{e}^{-\mathrm{i}\omega(r-1)\cdot\tau_1(\theta_k)} \tag{11}$$

where $\tau_{r,k}$ is the delay of the signal (of source k) received at sensor r relative to the first sensor.

Therefore, the array's input vector is

$$\boldsymbol{x}(f) = \mathrm{A} \cdot \boldsymbol{s}(f) + \boldsymbol{n}(f) \tag{12}$$

Fig. 2 shows the conditions of the proposed scenario in which the desired and noise signals are jointly sub-Gaussian as given by (9).

4 Maximum Likelihood Estimator

The observed signal $\boldsymbol{x} = [x_1 \ldots x_\rho]^{\mathrm{T}}$ is of the form $\boldsymbol{x}_r(t) = y(t)^{1/2} \cdot \boldsymbol{z}_r(t)$ where, as the transmitted one, the received signal is sub-Gaussian. It is therefore straightforward to show that the received signal's $\boldsymbol{z}$ statistics will be relating to those of the transmitted signal $\boldsymbol{v}$ by:

$$\mathrm{R} = \mathrm{A}\,\Sigma_v\,\mathrm{A}^{\dagger} + \sigma_n^2 \mathrm{I}_\rho \tag{13}$$

Therefore, the maximum likelihood estimator is

$$[\hat{\Sigma}, \hat{\boldsymbol{\theta}}] = \arg\max_{\hat{\Sigma},\hat{\boldsymbol{\theta}}} \prod_{f=f_1}^{f_M} \frac{\left[\boldsymbol{x}^{\dagger}(f)\mathrm{R}^{-1}\boldsymbol{x}(f) + \frac{1}{4}\right]^{-1}}{2\sqrt{\pi}\,\pi^{\rho}\,|\mathrm{R}|} \tag{14}$$

and simplified by taking the $\log_e$

$$\begin{aligned} [\hat{\Sigma}, \hat{\boldsymbol{\theta}}] = \arg\min_{\hat{\Sigma},\hat{\boldsymbol{\theta}}} \sum_{f=f_1}^{f_M} \Big\{ & \log_e |\mathrm{R}| \\ & + \log_e \left[\boldsymbol{x}^{\dagger}(f)\mathrm{R}^{-1}\boldsymbol{x}(f) + {}^{1}\!/_{4}\right] \Big\} \end{aligned} \tag{15}$$

5 Separable Solution

We proceed in this case to reach an alternative minimization function both to reduce the search space and to avoid the numerical errors introduced in the first term when the matrix is not full rank.

5.1 Estimating the Statistics

We follow the same procedure as in [6] where the ML function is minimized with respect to the signal statistics assuming known DOA.

Summation is omitted for the derivations, and thus we define the function to be minimized as:

$$\mathcal{L} = \underbrace{\log_e |\mathrm{R}|}_{\mathcal{L}_1} + \underbrace{\log_e \left[\boldsymbol{x}^\dagger(f)\mathrm{R}^{-1}\boldsymbol{x}(f) + {}^1\!/_4\right]}_{\mathcal{L}_2} \quad (16)$$

Differentiating:

$$\frac{\partial \mathcal{L}_1}{\partial \sigma_{ij}} = \frac{\partial \log_e |\mathrm{R}|}{\partial \sigma_{ij}} = \mathrm{Tr}\left[\mathrm{R}^{-1}\boldsymbol{a}_i\boldsymbol{a}_j^\dagger\right] = \boldsymbol{a}_j^\dagger \mathrm{R}^{-1}\boldsymbol{a}_i \quad (17)$$

Similarly, for the second term:

$$\frac{\partial \mathcal{L}_2}{\partial \sigma_{ij}} = \frac{\partial \log_e \left[\boldsymbol{x}^\dagger(f)\mathrm{R}^{-1}\boldsymbol{x}(f) + {}^1\!/_4\right]}{\partial \sigma_{ij}} = -\frac{1}{\mathrm{Tr}\left[\mathrm{R}^{-1}\mathrm{C}\right] + {}^1\!/_4} \cdot \boldsymbol{a}_j^\dagger \mathrm{R}^{-1}\,\mathrm{C}\,\mathrm{R}^{-1}\,\boldsymbol{a}_i \quad (18)$$

where we define $\mathrm{C} = \boldsymbol{x}\boldsymbol{x}^\dagger$.

Therefore

$$\frac{\partial \mathcal{L}}{\partial \sigma_{ij}} = \boldsymbol{a}_j^\dagger \left[\mathrm{R}^{-1} - \frac{\mathrm{R}^{-1}\,\mathrm{C}\,\mathrm{R}^{-1}}{\mathrm{Tr}\left[\mathrm{R}^{-1}\mathrm{C}\right] + {}^1\!/_4}\right]\boldsymbol{a}_i \quad (19)$$

or in matrix notation

$$\frac{\partial \mathcal{L}}{\partial \Sigma} = \mathrm{A}^\dagger \mathrm{R}^{-1}\left[\mathrm{R} - \frac{\mathrm{C}}{\mathrm{Tr}\left[\mathrm{R}^{-1}\mathrm{C}\right] + {}^1\!/_4}\right]\mathrm{R}^{-1}\mathrm{A}_i \quad (20)$$

Using the *Sherman-Morrison-Woodbury identity*

$$\left(\mathrm{A} + \mathrm{U}\,\mathrm{V}^\dagger\right)^{-1} = \mathrm{A}^{-1} - \mathrm{A}^{-1}\mathrm{U}\left(\mathrm{I} + \mathrm{V}^\dagger\,\mathrm{A}^{-1}\,\mathrm{U}\right)^{-1}\mathrm{V}^\dagger\,\mathrm{A}^{-1} \quad (21)$$

with relation (13) connecting R to the original signal statistics Σ

$$\mathrm{R}^{-1} = \frac{1}{\sigma_n^2}\left\{\mathrm{I} - \mathrm{A}\left(\Sigma\,\mathrm{A}^\dagger\,\mathrm{A} + \sigma_n^2\mathrm{I}\right)^{-1}\Sigma\,\mathrm{A}^\dagger\right\} \quad (22)$$

and

$$\mathrm{R}^{-1}\mathrm{A} = \mathrm{A}\left\{\Sigma\,\mathrm{A}^\dagger\,\mathrm{A} + \sigma_n^2\mathrm{I}\right\}^{-1} \quad (23)$$

Substituting back in eq. (20)

$$\frac{\partial \mathcal{L}}{\partial \Sigma} = \left\{\Sigma\,\mathrm{A}^\dagger\,\mathrm{A} + \sigma_n^2\mathrm{I}\right\}^{-1}\mathrm{A}^\dagger\left[\mathrm{R} - \frac{\mathrm{C}}{\mathrm{Tr}\left[\mathrm{R}^{-1}\mathrm{C}\right] + {}^1\!/_4}\right]\mathrm{A}\left\{\Sigma\,\mathrm{A}^\dagger\,\mathrm{A} + \sigma_n^2\mathrm{I}\right\}^{-1}$$

But $\frac{\partial \mathcal{L}}{\partial \Sigma} = 0$, therefore

$$\mathrm{A}^\dagger\left[\mathrm{A}\,\Sigma_v\,\mathrm{A}^\dagger + \sigma_n^2\mathrm{I} - \frac{\mathrm{C}}{\mathrm{Tr}\left[\mathrm{R}^{-1}\mathrm{C}\right] + {}^1\!/_4}\right]\mathrm{A} =$$

$$\mathrm{A}^\dagger\mathrm{A}\,\Sigma_v\,\mathrm{A}^\dagger\mathrm{A} + \mathrm{A}^\dagger\,\sigma_n^2\,\mathrm{A} - \frac{\mathrm{A}^\dagger\,\mathrm{C}\,\mathrm{A}}{\mathrm{Tr}\left[\mathrm{R}^{-1}\mathrm{C}\right] + {}^1\!/_4} = 0 \quad (24)$$

Solving for Σ, we can find the Σ_{ML} (over all available data):

$$\Sigma_{\mathrm{ML}} = \sum_{t=t_1}^{t_M}\left[\mathrm{A}^-\left(\frac{\boldsymbol{x}\boldsymbol{x}^\dagger}{\mathrm{Tr}\left[\mathrm{R}^{-1}\boldsymbol{x}\boldsymbol{x}^\dagger\right] + {}^1\!/_4} - \sigma_n^2\right)\mathrm{A}^{-\dagger}\right] \quad (25)$$

where in the above $\mathrm{A}^- = \left(\mathrm{A}^\dagger\mathrm{A}\right)^{-1}\mathrm{A}^\dagger$ and R^{-1} as defined in (26). The solution of (25) can be easily found using the numerical iteration method.

$$\mathrm{R}^{-1} = \frac{1}{\sigma_n^2}\left\{\mathrm{I} - \mathrm{A}\left(\Sigma\,\mathrm{A}^\dagger\,\mathrm{A} + \sigma_n^2\mathrm{I}\right)^{-1}\Sigma\,\mathrm{A}^\dagger\right\} \quad (26)$$

An estimate of R can be used as an initial guess and can be found from the data using a covariation measure. Alternatively, as we have observed, a *random or identity* initialization matrix also allows for fast convergence. Similarly the noise variance σ_n^2 can also be found from the same covariation measure assuming the number of sources and sensors are known, similarly to the Gaussian case [6, 7].

5.2 DOA Estimation

Clearly, the above assumes that the DOA vector is known, *and here we approach the localization part of the problem*. Using a pseudo-ML approach we can express the modified ML function irrespective of the statistics R as

$$\hat{\boldsymbol{\theta}} = \arg\min_{\hat{\boldsymbol{\theta}}} \sum_{f=f_1}^{f_M}\left\{\log_e\left[\boldsymbol{x}^\dagger(f)\mathrm{R}^{-1}\boldsymbol{x}(f) + {}^1\!/_4\right]\right\} \quad (27)$$

where R can be substituted with any valid statistics (identity matrix, for instance). A search algorithm, or an alternating line search approach, can be used to find the solution of the above equation. The optimum solution can be reached by recursively iterating between (25) and (27).

6 DOA Simulations

Simulations in this section are performed using a narrowband signal, *where we assume that d, the intersensor distance, is equal to* $\lambda/2$. *In the following four cases,* we had random DOA's for 2 sources, 8 sensors, and blocks of 32 samples, and $\Sigma = \mathrm{I}$ is assumed to hold, although the test matrix had a random correlation structure, but always with diagonal elements of dispersion equal to the dispersion of the Lévy sequence ($\gamma_s = \gamma_u = \gamma_v = 1$). Additionally the impulsiveness was kept constant ($\alpha = 1$ for cases 1 & 4, and $\alpha = 2$ for 2 & 3 as described below). The Generalized Signal-to-Noise Ratio used below is defined as:

$$\mathrm{GSNR} = 10\log_{10}\left(\frac{\gamma_s}{\gamma_n}\right) = -10\log_{10}(\gamma_n) \quad (28)$$

The four simulation scenarios were:

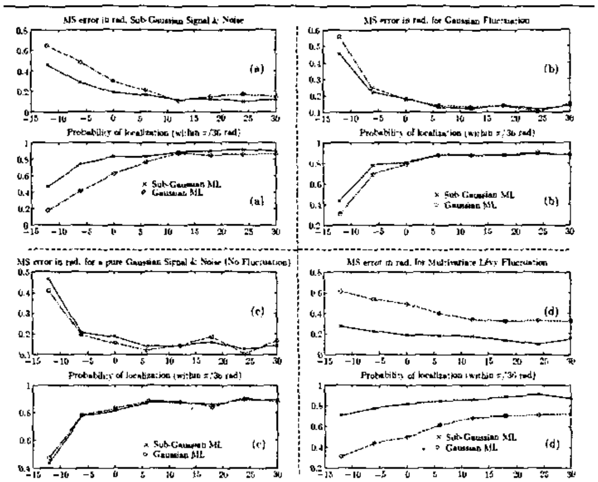

Figure 3: Simulations demonstrate the obtained benefit in localization by using the Stochastic ML method based on the Lévy Sub-Gaussian processes versus the Gaussian ML method for the conditions described in the text. Robustness of the Sub-Gaussian method is apparent especially in case (d).

1. Exactly as per the derivation assumptions (Fig. 3a): received signal is sub-Gaussian, created from a Multivariate Gaussian and a univariate Lévy. Received signal impulsiveness is $\alpha = 1$
   (impulsiveness – dependence)

2. The signal is a Multivariate Gaussian (Fig. 3b), and is created from a Multivariate Gaussian ($\boldsymbol{v}$) and a univariate Gaussian (w). Received signal impulsiveness is $\alpha = 2$
   (no impulsiveness – dependence)

3. The signal is a Multivariate Gaussian (Fig. 3c) and it undergoes *no* energy fluctuation ($w = 1$, $\boldsymbol{v} = \boldsymbol{s}$). This conforms to the assumptions of the well known Gaussian based ML. Clearly, the received signal impulsiveness is $\alpha = 2$
   (no impulsiveness – no dependence)

4. Finally, the received signal is sub-Gaussian(Fig. 3d), created from a Multivariate Gaussian ($\boldsymbol{v}$) and a *Multivariate* Lévy ($\boldsymbol{w}$). In this case, the signals can be viewed as simply Cauchy. Received signal impulsiveness is $\alpha = 1$
   (impulsiveness – no dependence)

7 Experiments

In order to test the localization algorithm with some real data, we constructed two synthetic microphone arrays: using the 10.2 channel system and ProTools we played back several (dry) signals. These audio channels were played together in various combinations through the loudspeakers at 48kHz, and 2 microphones were shifted forming a linear array. The synchronized playback–recording feature of ProTools, confirmed by the addition of chirp synchronization signals at the start of the recording, ensured that the array was accurately created (the second array more so than the first). The DOA of the sound signals was then estimated with both the Gaussian based ML and the sub-Gaussian based ML.

The ML functions for the following cases were evaluated over all frequencies by re-calculating the transformation matrix A for all possible $(\boldsymbol{\theta}, f)$ combinations, which is a computationally expensive process. A *Non-linear FFT* (NFFT) was used in order to keep the resulting frequency bands of the signals narrow. Specifically, we employed the method described by Mitra *et al* in [8] with a first order all-pass filter and a 30ms window (1440 samples). A visual representation of the first-order mapping is shown on Fig. 4 for a filter with fewer taps than the one actually used.

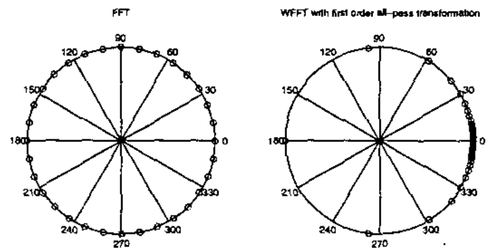

Figure 4: Sample frequency transformation of a 1st order all-pass filter

	Gaussian	Sub-Gaussian
45° angle RMS error	16	5
90° angle RMS error	22.5	10
Overall RMS error	19.5	8

Table 1: Errors for the Gaussian based ML method are more than double those of the sub-Gaussian based ML.

7.1 20-Microphone Array

In the 20-microphone array case, the aperture was 38cm with an intersensor spacing of 2cm, while two originally dry signals (Trumpet and Cello) were transmitted. Although this array was not very accurately spaced, there was a direct line of sight between the loudspeakers and the array.

Results of localization demonstrate that the sub-Gaussian based ML method performs significantly better than its Gaussian counterpart. Fig. 5 shows 7s of the signal where the cello and trumpet are being played. The sources were placed at 45° and at 90°. Each frame used for localization corresponds to a sliding window of 30ms. As can be observed, the sub-Gaussian ML method works significantly better. Table 1 shows the RMS error for this localization experiment and reveals that the performance of the Gaussian based ML is significantly worse than that of the sub-Gaussian based ML.

7.2 41-Microphone Array

In the 41-microphone array the recording conditions are similar to the previous array. However, the array spacing is 1cm and the sources are two speech signals (a female voice in English, and a female voice in Danish) placed at 48° and 110°. In addition, the arrangement is such that a strong echo is created at 90° and the 110° is significantly masked by the console, while the 48° source faces towards the console and thus creates further strong indirect paths to the array. Fig. 6 shows how the sources and array are positioned, as well as the flat screen, which as we expect caused a strong reflection. The RMS error of localization for the two methods is shown on Table 2.

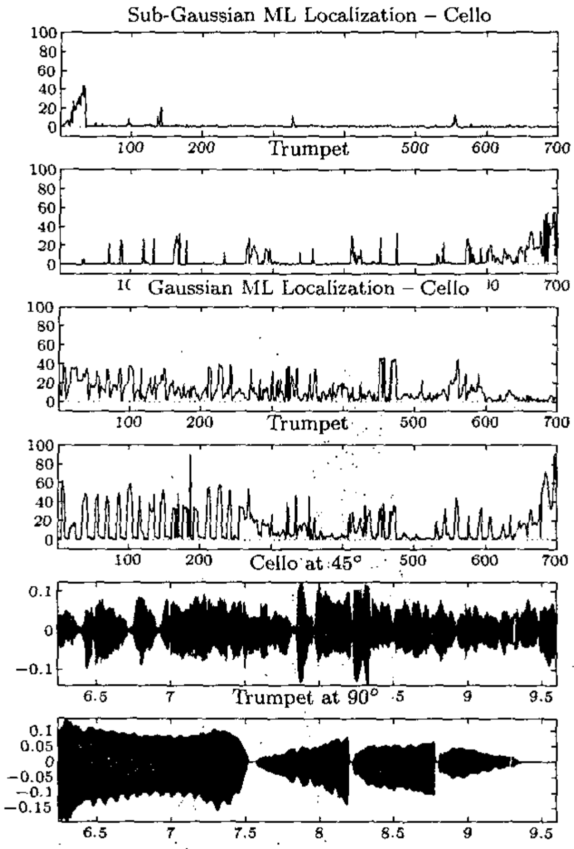

Figure 5: Error versus block sample for the localization of the two sources at 45° and at 90° using the sub-Gaussian and Gaussian ML based methods. The Gaussian based ML seems to suffer significantly from reverberation effects. The two original sound signals are plotted at the bottom two graphs, and we can see the correlation of the error rising when the amplitude of the trumpet dies off at the end.

8 Conclusions

We have presented a model that is able to model two significant features of real world audio signals, namely the impulsiveness of sound and the dependence of reverberation on the original source. We have derived the density of this model and presented the ML separable solution based on this density function of an array signal processing problem.

The proposed model and arrangement are tested in real world conditions by localizing 2 pairs of signals, a music pair and a speech pair. In both instances, the ML algorithm based on the proposed model significantly outperforms the Gaussian based ML method.

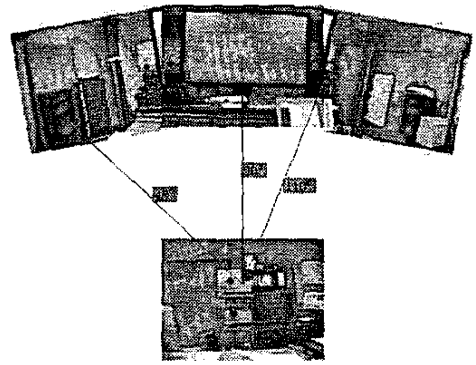

Figure 6: Arrangement of 41-microphone array. The microphone array was placed directly opposite the screen, where the projector is normally located (the projector was removed).

	Gaussian	Sub-Gaussian
48° angle RMS error	11.1	9.3
90° angle RMS error	13.1	6.9
110° angle RMS error	17.7	6.6
Overall RMS error	24.6	13.3

Table 2: Errors for the Gaussian based ML method are much higher than those of the sub-Gaussian based ML, but compare better under these conditions of the larger array than in the case of the 20-microphone array.

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